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A Channel Estimation Algorithm for MIMO-SCFDE

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Abstract—This letter proposes a novel method for channel estimation in a single-carrier multiple input-multiple output (MIMO) system with frequency-domain equalization/detection. To this end, we construct novel short MIMO training sequences that have constant envelope in the time domain to preclude the peak-to-average power ratio problem encountered in many systems that utilize the frequency domain for data recovery. Simultaneously, the spectrum in the frequency domain is flat except for a grid of nulls for predefined frequency tones. Armed with these sequences, we provide an algorithm that is optimal in the least squares (LS) sense at a potentially low computational cost. Results show that the algorithm performs identically to other proposed LS techniques. Furthermore, the algorithm is extremely bandwidth efficient in that the total training overhead required to obtain full CSI is just one block.

Index Terms—Channel estimation, MIMO, OFDM, SCFDE.

I. INTRODUCTION

VERY popular low-complexity broadband technique that has seen extensive research in the past decade is orthogonal frequency-division multiplexing (OFDM). Lately, however, much attention has been focused on another broadband technique, namely single-carrier (SC) transmission with frequency-domain equalization (FDE). SCFDE systems promise equal complexity and performance to OFDM systems without the high peak-to-average power ratio (PAPR) problem that plagues OFDM systems [1], [2]. Although channel estimation in OFDM systems has been studied extensively in both the single and multiantenna cases (e.g., see [3] and references therein), this topic has been left relatively unaddressed for SCFDE systems.

Typically, multiple input-multiple output (MIMO) channels are estimated by transmitting orthogonal training sequences in conjunction with least squares (LS) or minimum mean-square error (MMSE) methods. In [4], a recursive reconstructive (RR) channel estimation method based on novel training sequences was proposed for MIMO-SCFDE systems. In this letter, we eliminate the recursive process described in [4], thereby reducing the complexity of estimating the channel while maintaining bandwidth efficiency. For most practical cases, the algorithm is in fact a computationally efficient method of meeting the LS criterion.

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II. SYSTEM AND CHANNEL DESCRIPTION

Consider a wideband MIMO-SCFDE system with n_T transmit antennas and n_R receive antennas in which a length- N baseband sequence is modulated onto a SC waveform at each transmit antenna for transmission across a wireless channel. The received baseband sequences are equalized in the frequency domain. A cyclic prefix is added to each sequence prior to transmission and removed from each of the received sequences. We assume that the cyclic prefix is greater than or equal to the channel memory order, thereby preventing interblock interference and aiding the equalization process.

Let \mathbf{t}_q be the length- N time domain training sequence transmitted from antenna q , and let $\mathbf{h}_{p,q}$ be a length- N column vector that denotes the complex frequency response of the channel between transmit antenna q and receive antenna p . We denote the channel coefficient on the n th tone by $h_{p,q;n}$. The entire channel can be represented by the vector $\mathbf{h} = [\mathbf{h}_{1,1}^T, \mathbf{h}_{1,2}^T, \dots, \mathbf{h}_{n_R,n_T}^T]^T$.

An N -point discrete Fourier transform (DFT) is employed at each receive antenna in a MIMO-SCFDE system prior to equalization. In the frequency domain, the received signal at the p th antenna on the n th tone can be mathematically described by

$$y_{p;n} = \sum_{q=1}^{n_T} h_{p,q;n} x_{q;n} + \eta_{p;n} \quad (1)$$

where $x_{q;n}$ denotes the n th tone of the training sequence transmitted from the q th antenna and the noise term $\eta_{p;n} \sim \mathcal{CN}(0, \sigma^2)$ ¹. As shown in (1), the signal received at the p th antenna is a superposition of all of the transmitted training signals, which complicates symbol detection and training. For data detection, the received signal is equalized and transformed back into the time domain with an inverse DFT (IDFT). For channel estimation, however, the proposed algorithm can be employed following the DFT as described in the next section.

III. CHANNEL ESTIMATION FOR MIMO-SCFDE

The advantage of using this algorithm is that it allows the transmitted sequence to be nulled on certain frequency tones, causing the transmitted training sequences to be orthogonal in the frequency domain. Thus, (1) reduces to

$$y_{p;n} = h_{p,q;n} x_{q;n} + \eta_{p;n}, \quad n \in \Omega_q \quad (2)$$

where Ω_q is the set of tones over which training data is transmitted from the q th antenna. The nulled tones are then reconstructed at the receiver to provide a full channel estimate by executing the following steps for each channel path to reconstruct the entire frequency response:

¹ \mathcal{CN} denotes the complex normal distribution.

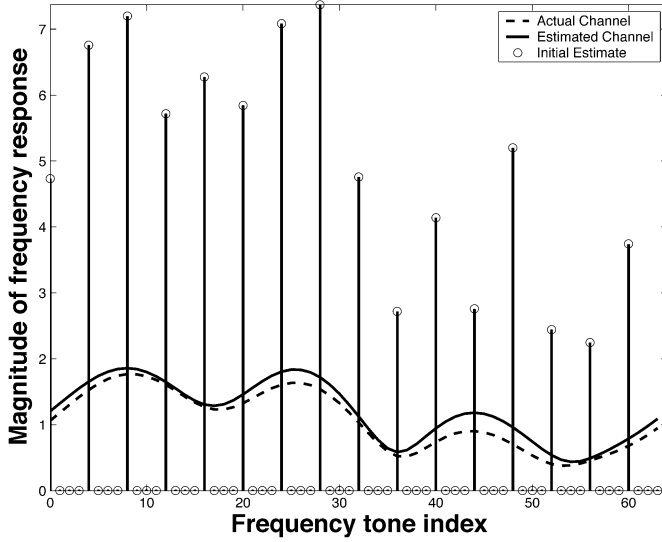


Fig. 1. Illustration of the algorithm for a single channel in a MIMO system where $n_T = 4$, $N = 64$, and SNR = 10 dB.

- 1) obtain initial channel estimate;
- 2) multiply initial channel estimate by n_T ;
- 3) convert channel estimate into the time domain and window significant taps;
- 4) convert this time domain signal back into the frequency domain.

The initial channel estimate $\hat{h}_{p,q;n}^{(0)}$ can be obtained by simply dividing the received signals on the tones in Ω_q by the unique transmitted training signals, giving

$$\hat{h}_{p,q;n}^{(0)} = \begin{cases} \frac{y_{p;n}}{x_{q;n}}, & n \in \Omega_q \\ 0, & n \notin \Omega_q. \end{cases} \quad (3)$$

The simple step of scaling the initial channel estimate by a factor of n_T can be physically interpreted as a means of recovering the original amount of energy in the channel that was removed by nulling certain frequency tones during transmission. If the pilot tones are evenly spaced, the total amount of energy is appropriately redistributed across the entire spectrum by the IDFT and DFT operations as seen in Fig. 1, [5]. The final channel estimate for all channel paths, $\hat{\mathbf{h}}$ is given by

$$\hat{\mathbf{h}} = \mathcal{K} \hat{\mathbf{h}}^{(0)} \quad (4)$$

where $\mathcal{K} = \text{diag}\{\mathbf{K}_{1,1}, \dots, \mathbf{K}_{n_R, n_T}\}$ and $\hat{\mathbf{h}}^{(0)}$ is the initial channel estimate. The $N \times N$ matrix $\mathbf{K}_{p,q}$ is defined by

$$\mathbf{K}_{p,q} = n_T \mathbf{F} \mathbf{W}_{T,p,q} \mathbf{F}^{-1} \quad (5)$$

where \mathbf{F} is the $N \times N$ normalized DFT matrix and $\mathbf{W}_{T,p,q}$ is an $N \times N$ rectangular windowing matrix with 1's on the main diagonal, where time delay taps exist and zeros are elsewhere for the channel path between transmit antenna q and receive antenna p . Note that $\mathbf{K}_{p,q}$ will be the same for all p and q if the channel length is assumed the same for all channel paths. A similar result for OFDM systems was presented in [6], [7].

As stated earlier, it is essential that the training sequences are designed such that the pilot tones are equally distributed across the entire bandwidth. The design of training sequences

with these properties is addressed in Section IV. Also, it is assumed that the length of the channel is known. If this assumption does not hold, the RR method presented in [4] can still be implemented to obtain a good channel estimate.

A. Mean Square Error (MSE)

Define the matrix \mathbf{W}_q as a matrix with 1's on the main diagonal indexes where pilot symbols are placed and $\mathbf{W} = \text{diag}\{\mathbf{W}_1, \dots, \mathbf{W}_q\}$. Rewriting the initial channel estimate as $\hat{\mathbf{h}}^{(0)} = \mathbf{W}(\mathbf{h} + \tilde{\eta})$, it can be shown that the mean square error (MSE) of this algorithm is

$$\text{MSE} \propto \text{tr} \left\{ (\mathbf{I} - \mathcal{K} \mathbf{W}) \mathbf{R} (\mathbf{I} - \mathcal{K} \mathbf{W})^H + \sigma_{\tilde{\eta}}^2 \mathcal{K} \mathbf{W} \mathbf{W}^H \mathcal{K}^H \right\} \quad (6)$$

where $\tilde{\eta}_{p;n} = \eta_{p;n}/x_n \forall n \in \Omega_q$, $\mathbf{R} = \mathcal{E}\{\mathbf{h} \mathbf{h}^H\}$ and $\sigma_{\tilde{\eta}}^2 \mathbf{I} = \mathcal{E}\{\tilde{\eta} \tilde{\eta}^H\}$. Equation (6) shows that both noise and windowing deficiencies contribute to the estimation error.

IV. TRAINING SEQUENCE DESIGN

As previously mentioned, the intentional nulling of frequency tones in the transmitted training sequences is a key requirement for bandwidth efficiency in using the proposed algorithm. One such training sequence can be constructed for transmission from the first antenna by simply repeating any arbitrary length- (N/n_T) sequence, denoted by $\bar{\mathbf{t}}_1 \triangleq (\bar{t}_{1,0}, \dots, \bar{t}_{1,N/n_T-1})$, n_T times to give $\mathbf{t}_1 \triangleq (t_{1,0}, \dots, t_{1,N-1})$. For this method, N must be evenly divisible by n_T . The n th tone of the DFT of \mathbf{t}_1 is given by

$$x_{1;n} = \begin{cases} \chi_{1;n}, & n \bmod n_T = 0 \\ 0, & \text{otherwise} \end{cases} \quad (7)$$

where $\chi_{1;n} \in \mathbb{C}$. Equation (7) follows from a property of repeated finite-length sequences [8] and suggests that the set of nonzero frequency tones for the sequence \mathbf{t}_1 is $\Omega_1 = \{0, n_T, 2n_T, \dots, N - n_T\}$.

To refrain from transmitting from multiple antennas on a given frequency tone, the set of nonzero frequency tones Ω_q for a given sequence may be shifted for use by other antennas. This shifting is accomplished by progressively rotating the phase of the length- N time-domain training sequence such that the k th element of the sequence transmitted from the q th antenna for $q = 2, 3, \dots, n_T$ is given by

$$t_{q;k} = t_{1;k} e^{j2\pi k(q-1)/N}. \quad (8)$$

The result from the frequency shift property of DFTs [8] is as follows:

$$t_{1;k} e^{j2\pi k(q-1)/N} \Leftrightarrow x_{1;n-q+1}. \quad (9)$$

To avoid noise amplification on certain frequency tones, a desired property of the training sequences is a constant frequency-domain amplitude. This condition is satisfied by using a particular class of polyphase, constant-magnitude sequences known as Chu sequences as base training sequences [1], [9]. The k th element of a length- P Chu sequence is given by

$$c_k = \begin{cases} e^{j\pi r k^2/P}, & \text{for even } P \\ e^{j\pi r k(k+1)/P}, & \text{for odd } P \end{cases} \quad (10)$$

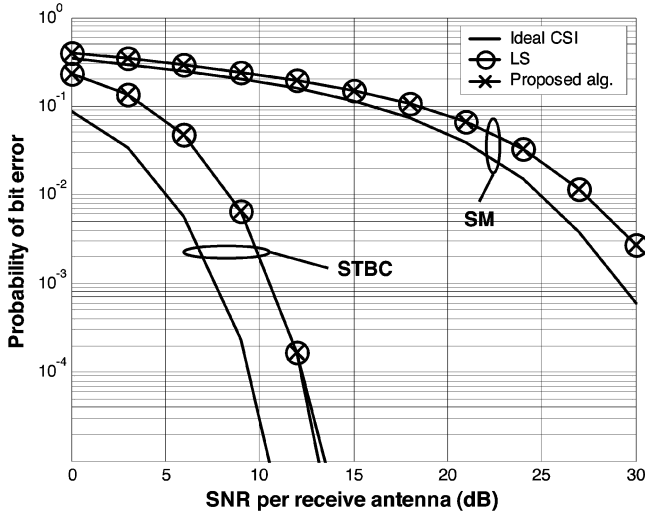


Fig. 2. Performance of STBC/SM-SCFDE systems. ($n_T = n_R = 4$, 16-QAM, 11-tap i.i.d. \mathcal{CN} channel, $N = 64$).

where r is relatively prime to P . Chu sequences were first studied for their good periodic autocorrelation properties. Here, we utilize the fact that the DFT of any arbitrary Chu sequence has a constant envelope. Therefore, the implementation of Chu sequences as base training sequences precludes the PAPR problem encountered in many FDE systems while minimizing the proposed algorithm's sensitivity to noise. It should be noted that although Chu sequences are polyphase sequences, they can be realized with direct digital synthesis (DDS) devices.

V. RESULTS AND DISCUSSION

The performance of the proposed algorithm was examined by simulating several different MIMO-SCFDE systems and observing the bit error rate for each system. All simulated systems utilized $n_T = 4$ transmit antennas and $n_R = 4$ receive antennas. For each system, 16-QAM symbols were partitioned into $N = 64$ symbol blocks. The blocks were then encoded according to one of two space-time processing techniques: spatial multiplexing (SM) [2] and space-time block codes (STBC) [10]. A cyclic prefix was appended to each block at the output of the space-time encoder to eliminate interblock interference caused by the channel. Each path of the fading MIMO channel was modeled as having 11 independent identically distributed (i.i.d.) complex Gaussian taps.

At the receiver, two methods of estimating the channel were employed for each system: the proposed method and an LS method as reported in [3], [11]. The referenced method was first proposed for MIMO-OFDM systems in [3] and is readily extendible to MIMO-SCFDE systems. The sequences described in Section IV were used in both channel estimation methods where only one block period was used for training. As a benchmark, one system was assumed to have perfect knowledge of the channel. A linear frequency-domain MMSE equalizer was implemented to recover the transmitted message. As illustrated in Fig. 2, the system employing the proposed method performs as well as the system using the method reported in [3].

Furthermore, the number of complex multiplications required to implement the proposed algorithm is $M_P = 2n_T n_R N \log_2 N + n_T n_R N$ whereas the complexity of

the method in [3] is given by $M_{[3]} = n_T^2 n_R L^2 + 2n_T n_R N L + n_T n_R N \log_2 N$ where L is the number of taps in the channel impulse response. It can easily be shown that for most practical systems (i.e., $64 \leq N \leq 1024$, $n_T \leq 16$), the proposed algorithm is less complex than the algorithm in [3] for channels with more than $L = 5$ symbols of dispersion.

As a final note, the performance of the algorithm degrades as the spacing of the pilot tones increases due to undersampling. The number of transmit antennas determines this spacing, as shown in Section IV. Therefore, we may use the Nyquist criterion to define an upper limit on the number of antennas that can transmit training data during any given block period as follows [11]:

$$n_T \leq \frac{NB_C}{B_{RF}} = \frac{N}{L} \quad (11)$$

where B_C is the coherence bandwidth and B_{RF} is the total RF bandwidth. If this limit is exceeded, time-domain aliasing of the estimated channel impulse response will occur.

VI. CONCLUSION

In this letter, we have developed a novel technique for channel estimation in MIMO wireless systems that utilize frequency-domain equalization/detection. The originality of the method is based on specially designed training sequences and the estimation algorithm. Specific construction of the training sequences combined with the time limited property of the wireless channel allows for efficient utilization of the bandwidth with minimal noise amplification during the estimation procedure. It was shown that the proposed method performs as well as the channel estimation technique given in [3], but benefits from a lower implementation complexity in virtually all cases of interest.

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